COMPUTATION OF THE RATE OF HELIUM EVAPORATION IN NITROGEN-FREE CRYOSTATS WITH HIGH-VACUUM INSULATION

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An approximate method is proposed for computing the rate of liquid helium evaporation and the temperature of the radiation shields in cryostats in which the shields are cooled by vapors of evaporating helium.

Cryostats with cooling of the radiation shields by vapors of the helium evaporating from the cryostat are most convenient for diverse deep-cooling researches [1-6]. The temperature of these shields is of crucial importance to the operation of a nitrogen-free cryostat. The rate of helium evaporation and the temperature of the shields were computed in [7] for Dewar vessels with high-vacuum insulation in which the heat flux to the helium tank occurs only because of radiation. Similar computations were later carried out for Dewar vessels with a multilayer vacuum shield insulation [8]. A method of computing the evaporation rate and the shield temperature for cryostats without nitrogen cooling and taking account of the heat fluxes most often encountered in actually operational devices is proposed below. It may well prove in

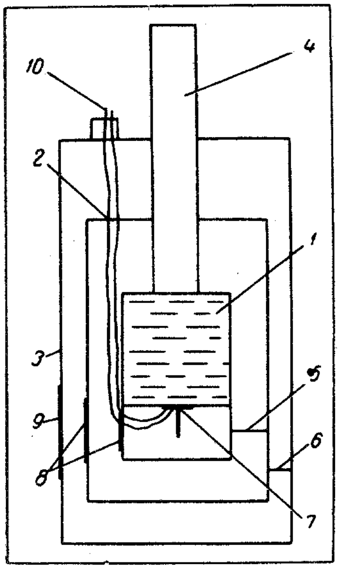

Fig. 1

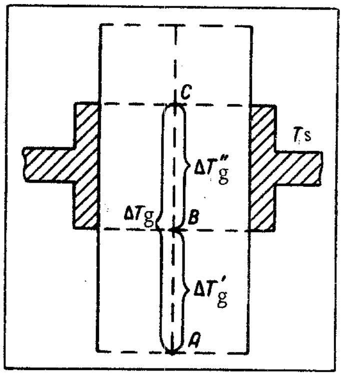

Fig. 2

Fig. 1. Schematic diagram of a nitrogen-free cryostat with one radiation shield. 1) Helium tank, 2) radiation shield, 3) outer housing, 4) suspension tube, 5) mechanical shield–tank coupling, 6) mechanical housing–shield coupling, 7) sample, 8) cooled optical filters, 9) entrance window, 10) electric wires.

Fig. 2. Cross section of the suspension tube.

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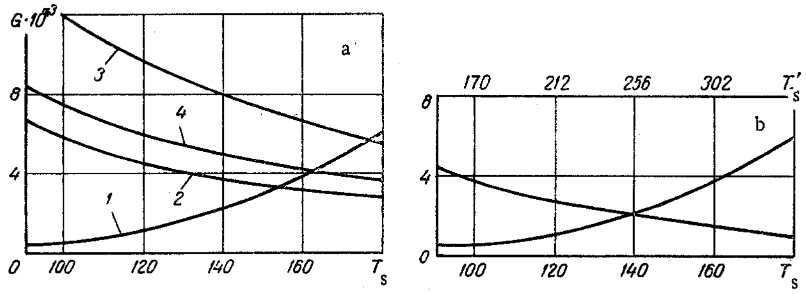

Fig. 3. Graphical analysis of a cryostat with $5 \cdot 10^{-4}$ m^3 capacity ($\varepsilon_{re_1} = 0.036$; $\varepsilon_{re_3} = 0.013$; $G \cdot 10^{-3}$, kg/h; T_s, °K): a) with one radiation shield, b) with two radiation shields.

due course that in low-capacity cryostats their magnitude is greater than the heat flux due to radiation.

The schematic diagram of such a cryostat is presented in Fig. 1. The radiation shield of this cryostat is cooled by vapors of the evaporating helium. Let us determine the heat fluxes delivered to the shield and the tank with helium from the outer walls of the cryostat calculated in the steady-state condition. Let us assume that there is no heat influx from the radiation shield to the helium tank and the outer housing to the shield by way of the suspension tube. This condition cannot be satisfied for sufficient sizes of the suspension tube and ideal heat exchange between the evaporating vapor and the inner surface of the suspension tube [9].

The heat from the outer housing to the radiation shield is transferred by radiation from the outer housing to the shield q_1 and external radiation background to the filter q_2, by heat conduction over the mechanical couplings between the housing and the shield q_3, and by heat conductivity over the electric wires q_4. Additional heating of the shield occurs because of evolution of Joule heat q_5 in the electric wires as the operating current flows through them (reliable heat contact between the wires and shield is assured). The heat fluxes mentioned can be determined from the following relationships [9-11]

$$q_1 = c_0 \varepsilon_{re1} F_1 \left[\left(\frac{T_h}{100} \right)^4 - \left(\frac{T_s}{100} \right)^4 \right], \tag{1}$$

where

$$\varepsilon_{re1} = \frac{1}{1/\varepsilon_1 + F_1/F_2 (1/\varepsilon_2 - 1)},$$

$$q_2 = c_0 \varepsilon_{re2} F_3 \left[\left(\frac{T_h}{100} \right)^4 - \left(\frac{T_s}{100} \right)^4 \right] \frac{\Omega}{2\pi} (1 - \tau), \tag{2}$$

and

$$\varepsilon_{re2} = \frac{1}{1/\varepsilon_3 + F_3/F_4 (1/\varepsilon_4 - 1)}.$$

This last expression has been written under the assumption that the filter is rigidly clamped to the shield and no signal is delivered to the sample from outside. In this case the radiation source is the background with the temperature of the outer housing T_h and the surface of the entrance window F_3:

$$q_3 = n_1 \frac{\lambda_1 S_1}{\Delta l_1} (T_h - T_s), \tag{3}$$

$$q_4 = n_2 \frac{\lambda_2 S_2}{\Delta l_2} (T_h - T_s). \tag{4}$$

Assuming for simplicity that all the Joule heat q_5 evolved in the wires is removed to the screen, it can then be considered that

$$q_5 = I^2 R_1 \Delta l_2. \tag{5}$$

TABLE 1. Comparison of Computed and Experimental Results

Cryostat capacity, 10^{-4} m^3	Surface area, 10^{-2} m^2			Computation		Experiment	
	housing	shield	tank	$G \cdot 10^{-3}$, kg/h	T_s, °K	$G \cdot 10^{-3}$, kg/h	T_s, °K
10	15,4	9,2	6,3	5,9	165	5,7	163
6	13,7	10,1	6,8	6,5	166	6,8	162
5	9,6	6,3	4,7	4,0	163	4,1	166
2,5	6,5	4,4	3,2	2,9	164	2,9	165

The total heat flux q^I to the radiation shield is defined in this case as the sum of the listed heat fluxes:

$$q^{I} = c_0 \left[\left(\frac{T_h}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]\left[\varepsilon_{re1} F_1 + \varepsilon_{re2} F_3 \frac{\Omega}{2\pi}(1-\tau)\right] + (T_h - T_s)\left(n_1 \frac{\lambda_1 S_1}{\Delta l_1} + n_2 \frac{\lambda_2 S_2}{\Delta l_2}\right) + I^2 R_1 \Delta l_2. \tag{6}$$

The heat flux q' heating the radiation shield is expended partially in direct evaporation of the liquid helium, and partially goes to the fastening of the radiation shield to the suspension tube and is cancelled by the evaporating cold vapor. The part q^I going from the shield directly into evaporation of the liquid helium is determined by the shield radiation to the helium tank q_6, by the heat conductivity over the mechanical couplings q_7 and the electrical wires q_8, and by radiation of the filter q_9. In addition, heat fluxes due to radiation of the entrance window q_{10} passing through the filter, radiation of the plug in the upper part of the cryostat throat q_{11}, and evolution of Joule heat by the sample q_{12} and the electrical wires q_{13} act on the tank with helium. These heat fluxes can be determined from the following relationships:

$$q_6 = c_0 \varepsilon_{re3} F_5 \left[\left(\frac{T_s}{100}\right)^4 - \left(\frac{T_g}{100}\right)^4\right], \tag{7}$$

where

$$\varepsilon_{re3} = \frac{1}{1/\varepsilon_5 + F_5/F_1\,(1/\varepsilon_1 - 1)},$$

$$q_7 = n_3 \frac{\lambda_3 S_3}{\Delta l_3}(T_s - T_g); \tag{8}$$

$$q_8 = n_2 \frac{\lambda_4 S_2}{\Delta l_4}(T_s - T_g); \tag{9}$$

$$q_9 = R_{T_s}^{\nu_1 - \nu_2} \frac{\Omega}{2\pi} F_3. \tag{10}$$

The required value $R_{T_s}^{\nu_1-\nu_2}$ can be found in tables [12].

The total heat flux q^{II} going into helium evaporation because of the factors considered above equals

$$q^{II} = c_0 \varepsilon_{re3} F_5 \left(\frac{T_s}{100}\right)^4 + (T_s - T_g)\left(n_3 \frac{\lambda_3 S_3}{\Delta l_3} + n_2 \frac{\lambda_4 S_2}{\Delta l_4}\right) + R_{T_s}^{\nu_1-\nu_2} \frac{\Omega}{2\pi} F_3. \tag{11}$$

The heat flux to the helium tank because of radiation from the window passing through the filter can be defined as:

$$q_{10} = c_0 \varepsilon_{re2} F_7 \left(\frac{T_h}{100}\right)^4 \frac{\Omega}{2\pi}\tau. \tag{12}$$

The heat flux to the helium because of plug radiation can be estimated from the expression

$$q_{11} = c_0 \varepsilon_{re4} F_6 \frac{\pi d^2}{8\pi l^2}\left(\frac{T_g}{100}\right)^4,$$

where

$$\varepsilon_{re4} = \frac{1}{1/\varepsilon_6 + (1/\varepsilon_7 - 1)} . \qquad (13)$$

Equation (13) is written down under the assumption that all radiation incident on the inner surface of the suspension tube is not absorbed completely therein. In this case, the radiation on the liquid helium surface is incident only in the small solid angle at which the lower end of the tube is seen from the upper. Radiation of the inner surfaces of the suspension tube is not taken into account here. The true value of the fraction of room radiation reaching the helium by the way of the tube can be found by a direct measurement in practice during assembly of the cryostat and will be close to the rated value if the tube walls are strongly oxidized on the inner surface.

Considering that all the Joule heat from the sample and wires in the section under consideration goes to the helium tank, we will have

$$q_{12} = I^2 R_3, \qquad (14)$$

$$q_{13} = I^2 R_2 \Delta l_4 n_2. \qquad (15)$$

Then the total heat flux q^{III} for evaporating the liquid helium in the cryostat is:

$$q^{III} = q^{II} + q_{10} + q_{11} + q_{12} + q_{13} = Gr. \qquad (16)$$

Part of the heat which is delivered by way of the shield to the place where it is fastened to the suspension tube q^{IV} is defined as the difference between the heat fluxes delivered to and from the shield

$$q^{IV} = q^{I} - q^{II}. \qquad (17)$$

It was assumed earlier that heat is not delivered to the liquid helium by way of the suspension tube, hence the flux q^{IV} should be cancelled because of the evaporating helium, i.e., the following relation should be maintained (Fig. 2):

$$q^{I} - q^{II} = Gc_p \Delta T_g. \qquad (18)$$

Manipulating the equations under consideration, we obtain

$$\frac{q^{I} - q^{II}}{q^{III}} = \frac{Gc_p \Delta T_g}{Gr} . \qquad (19)$$

The difference ΔT_g in this equation can be considered as the sum of the temperature differences $\Delta T_g'$ and $\Delta T_g''$, where $\Delta T_g'$ is the temperature difference to which the vapor from the entrance to the suspension tube to the lower part of the place where the shield is fastened is heated, and $\Delta T_g''$ is the temperature difference to which the vapor is heated while passing over the tube from the lower to the upper part of the shield fastening. In Fig. 2 these sections are between the points A and B, and B and C, respectively. Then

$$\Delta T_g = \Delta T_g' + \Delta T_g'' . \qquad (20)$$

The quantity ΔT_g in (20) is constrained by the second law of thermodynamics, from which it follows that the gas temperature at the point C, equal to (ΔT_g + 4.2), cannot be greater than the shield temperature. In the limit case (ΔT_g + 4.2) can equal T_s, however, the length of the suspension tube should then be quite large.

Assuming the gas temperature at the point C to differ from the shield temperature T_s by several degrees, it can be considered that

$$\Delta T_g = T_s - 4.2 - \delta. \qquad (21)$$

Taking the above into account, let us transform (19) relative to T_s. For T_g = 4.2°K, T_h = 300°K, we obtain

$$A\left(\frac{T_S}{100}\right)^5 - B\left(\frac{T_S}{100}\right)^4 + C\left(\frac{T_S}{100}\right)^2 - D\left(\frac{T_S}{100}\right) = E, \tag{22}$$

where

$$A = 6.25\cdot 10^5\,(\varepsilon_{re3}F_5 - 0.16\varepsilon_{re2}F_7\Omega\tau);$$

$$B = 4.96\cdot 10^2\,\varepsilon_{re3}F_5\,(1+12.6\delta) - 10^3\varepsilon_{re2}F_7\Omega\tau \times (4.2+\delta) - 2.58\cdot 10^4\left(n_3\,\frac{\lambda_3 S_3}{\Delta l_3} + n_2\,\frac{\lambda_4 S_2}{\Delta l_4}\right);$$

$$C = 1.26\cdot 10^4\left(n_3\,\frac{\lambda_3 S_3}{\Delta l_3} + n_2\,\frac{\lambda_4 S_2}{\Delta l_4}\right);$$

$$D = 10^2\Big[1.26(\delta+4.3)\left(n_2\,\frac{\lambda_4 S_2}{\Delta l_4} + n_5\,\frac{\lambda_3 S_3}{\Delta l_3}\right) - 0.2\,R_{T_S}^{\nu_1-\nu_2}\,\Omega F_3 - 6.3\cdot 10^4\left(1.28\varepsilon_{re2}F_7\Omega\tau + \varepsilon_{re2}F_6\,\frac{d^2}{l^2}\right) - 1.26I^2(R_3 + R_2\Delta l_4 n_2) - 5.2\left(n_1\,\frac{\lambda_1 S_1}{\Delta l_1} + n_2\,\frac{\lambda_2 S_2}{\Delta l_2}\right)\Big];$$

$$E = 2.08\cdot 10^6\,[\varepsilon_{re1}F_1 + 0.16\varepsilon_{re2}F_3\Omega(1-\tau)] + 1.56\cdot 10^3\left(n_1\,\frac{\lambda_1 S_1}{\Delta l_1} + n_2\,\frac{\lambda_2 S_2}{\Delta l_2}\right) + 5.2I^2R_1\Delta l_2 - 0.4\left(n_3\,\frac{\lambda_3 S_3}{\Delta l_3} + n_2\,\frac{\lambda_4 S_2}{\Delta l_4}\right)(1+13.25\delta) + 0.016R_{T_S}^{\nu_1-\nu_2}\,\Omega F_3\,(1+12.6\delta) + 0.8\cdot 10^5(\delta+4.2)\left(\varepsilon_{re2}F_7\Omega\tau + 0.79\varepsilon_{re4}F_6\,\frac{d^2}{l^2}\right) + 1.26I^2(\delta+4.2)(R_3 + R_2\Delta l_4 n_2).$$

A number of parameters in (22) depend functionally on T_S. This equation is solved for T_S by successive approximations applied to a specific designed instrument by selecting a suitable value of T_S. The losses by evaporation G for a known shield temperature are then defined as:

$$G = q^{III}/r. \tag{23}$$

For a cryostat in which the heat flux to the tank is only by radiation, (22) simplifies and takes a form similar to (3) in [7]:

$$A'\left(\frac{T_S}{100}\right)^5 - B'\left(\frac{T_S}{100}\right)^4 = E', \tag{24}$$

where

$$A' = 6.25\cdot 10^5\varepsilon_{re3}F_5;$$

$$B' = 2.6\cdot 10^4\,[\varepsilon_{re3}F_5\,(0.02+0.24\delta) - \varepsilon_{re1}F_1];$$

$$E' = 2.08\cdot 10^5\varepsilon_{re1}F_1.$$

The computation was verified on a series of similar cryostats of different capacities. After the rate of helium evaporation from the cryostats and the temperatures of their radiation shields had been meassured, the reduced radiativities of the working surfaces ε_{re_1} and ε_{re_3} were determined. They were found to equal 0.013 and 0.036. These values were inserted as initial values in the computation of other cryostats. Such cryostats were fabricated and investigated. The computed and experimental data (presented in Table 1) are in satisfactory agreement. The investigations conducted confirmed that, in practice, the heat flux over the suspension tube had no effect on the rate of helium evaporation from the cryostat.

Solutions for the shield temperature and the rate of helium evaporation can be obtained graphically. The following system of equations connecting the shield temperature to the rate of helium evaporation

$$Gr = c_0 \varepsilon_{re3} F_5 \left(\frac{T_s}{100}\right)^4,$$

$$Gc_p (T_s - 4{,}2 - \delta) = c_0 \varepsilon_{re1} F_1 \left[\left(\frac{T_h}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right] - c_0 \varepsilon_{re3} F_5 \left(\frac{T_s}{100}\right)^4, \quad (25)$$

can be composed on the basis of (11) and (18) for a cryostat in which the heat is supplied to the helium tank only by radiation from the radiation shield.

The results of a computation for a cryostat of $5 \cdot 10^{-4}$ m^3 capacity are presented in Fig.3a. The figure shows a family of curves for $T_h = 300$°K and $\delta = 3$, constructed for different ratios F_5/F_1 under the assumption that F_2 and F_5 are fixed and given. Curve 1 has been constructed from the first equation in the system, and 2, 3 and 4 from the second. Curve 2 corresponds to the limit case when the surface area of the radiation shield equals the tank surface area, and 3 corresponds to the other limit case when the shield surface area equals the housing surface area. Curve 4 corresponds to the selected construction. The ordinates and abscissae of the intersections of the curves yield G and T_s, respectively, for each of these cases. A change in the ratio F_5/F_1 from 0.5 to 1 results in a 20-25°K change in T_s in the 155-180°K temperature band. Hence the helium evaporability changes approximately 1.7-fold: An analogous problem taking account of the heat fluxes listed above is also solved graphically by successive approximation.

The relations presented can also be used to design analogous cryostats with two and more shields. To estimate the influence of the second shield on the evaporation rate and shield temperature, we considered a cryostat having the following parameters:

$$F_5 = 4.7 \cdot 10^{-2}\ \text{m}^2; \quad F_1 = 6.3 \cdot 10^{-2}\ \text{m}^2; \quad F_1' = 9.6 \cdot 10^{-2}\ \text{m}^2;$$

$$F_h = 13 \cdot 10^{-2}\ \text{m}^2; \quad \varepsilon_{re3} = 0.013; \quad \varepsilon_{re1} = 0.04; \quad \varepsilon_{re}' = 0.03.$$

The solution of the system of equations

$$q_{10} = c_0 \varepsilon_{re3} F_5 \left[\left(\frac{T_s}{100}\right)^4 - \left(\frac{T_g}{100}\right)^4\right] = Gr,$$

$$q_{21} = q_{10} + Gc_p (T_s - 4.2 - \delta) = c_0 \varepsilon_{re}' F_1 \left[\left(\frac{T_s'}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right], \quad (26)$$

$$q_{32} = q_{21} + Gc_p (T_s' - T_s) = c_0 \varepsilon_{re1} F_1' \left[\left(\frac{T_h}{100}\right)^4 - \left(\frac{T_s'}{100}\right)^4\right]$$

was found graphically.

The following relations were obtained from (26) by manipulation:

$$G = \frac{c_0 \varepsilon_{re3} F_5 \left(\frac{T_s}{100}\right)^4}{r},$$

$$G = \frac{c_0 \varepsilon_{re1} F_1' \left[\left(\frac{T_h}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]}{r + c_p (T_s - 4.2 - \delta)}, \quad (27)$$

$$T_s' = T_s \left\{ \frac{\varepsilon_{re3} F_5}{\varepsilon_{re}' F_1} \left[1 + \frac{c_p (T_s - 4.2 - \delta)}{r}\right] + 1 \right\}^{1/4}.$$

The results of a computation for a cryostat of capacity $5 \cdot 10^{-4}$ m^3 are presented in Fig.3b, where the temperature of the first shield in °K is plotted along the abscissa according to the lower scale, and that of the second shield according to the upper scale, and the rate of evaporation in kg/h is plotted along the ordinate. The appropriate value of evaporability can be found at the point of intersection of the curves. Use of a second shield lowers the temperature of the first by 20-25°K and diminishes the rate of helium evaporation 1.8-1.9-fold as compared with a single-shield cryostat. From the computation it appears that the temperature of the first cryostat shield cannot be higher than 160°K for the selected values of the radiativities of the working surfaces. The dependences presented in Fig.3a,b prove to be valid even for cryostats of different capacities.

However, it should not be forgotten that the lower rate of helium evaporation is achieved in practice because of an increase in the cryostat size and weight.

NOTATION

c_0	radiation coefficient of an absolutely black body;
$\varepsilon_{re_1}, \varepsilon_{re_2}, \varepsilon_{re_3}, \varepsilon_{re_4}$	reduced emissivities respectively, of the housing and shield, entrance background and filter, shield and helium tank, plug and liquid helium;
$F_1, F_2, F_3, F_4, F_5, F_6, F_7$	surface areas, respectively, of the shield, housing, filter, window, tank (flask), liquid helium, and sample;
T_h, T_s, T_g	temperature of housing, shield, and tank surfaces;
$\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_7$	emissivities, respectively, of the shield, housing, filter, window, tank, plug, and liquid helium surfaces;
Ω	solid angle within which the window radiation is distributed;
τ	filter transmission coefficient;
n_1, n_2, n_3	the number of similar mechanical couplings and electrical wires between the housing and shield and shield and tank;
$\lambda_1, \lambda_2, \lambda_3, \lambda_4$	mean heat conductivity of the mechanical couplings and electrical wires in appropriate temperature bands;
S_1, S_2, S_3	cross-sectional areas, respectively, of the mechanical housing–shield coupling, the wires, and the mechanical shield–tank couplings;
$\Delta l_1, \Delta l_2, \Delta l_3, \Delta l_4$	lengths, respectively, of the mechanical couplings and wires in the housing–shield and shield–tank sections;
I	working current;
R_1, R_2, R_3	mean resistivities, respectively, of wires in the housing–shield and shield–tank sections, and sample resistivity at the working temperature;
l	total length of the suspension tube;
d	diameter of the suspension tube;
$R_{T_s}^{\nu_1-\nu_2}$	radiation of the filter surface in the wavelength band from ν_1 to ν_2 at the temperature T_s
G	mass flow rate of the evaporating helium per unit time;
r	latent heat of vapor-formation of helium;
ΔT_g	temperature difference to which the vapor is heated from the entrance in the suspension tube to the upper part of the place where the shield is fastened to the tube;
δ	temperature difference between the shield and the vapor at the point C;
c_p	specific heat of the helium gas;
F_1', T_s'	surface area and temperature of the second shield in a cryostat with two shields;
ε_{re}'	reduced emissivity of the first and second shield surfaces;
q_{10}, q_{21}, q_{32}	heat fluxes in the cryostat with two shields from the first shield to the tank, from the second shield to the first, and from the housing to the second shield, respectively.

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